GRASP Laboratory
General Robotics & Autonomous Systems and Processes

Mechatronic Modeling and Design with Applications in Robotics

Introduction to Modeling

Modeling

1. What is specifically meant by modeling?
2. What is the use of models? Input → Output
3. What types of modeling are possible?
4. How to model a dynamic system?

A way to develop an analytical model that has the four characteristics: Integrated, Unified, Unique and Systematic.

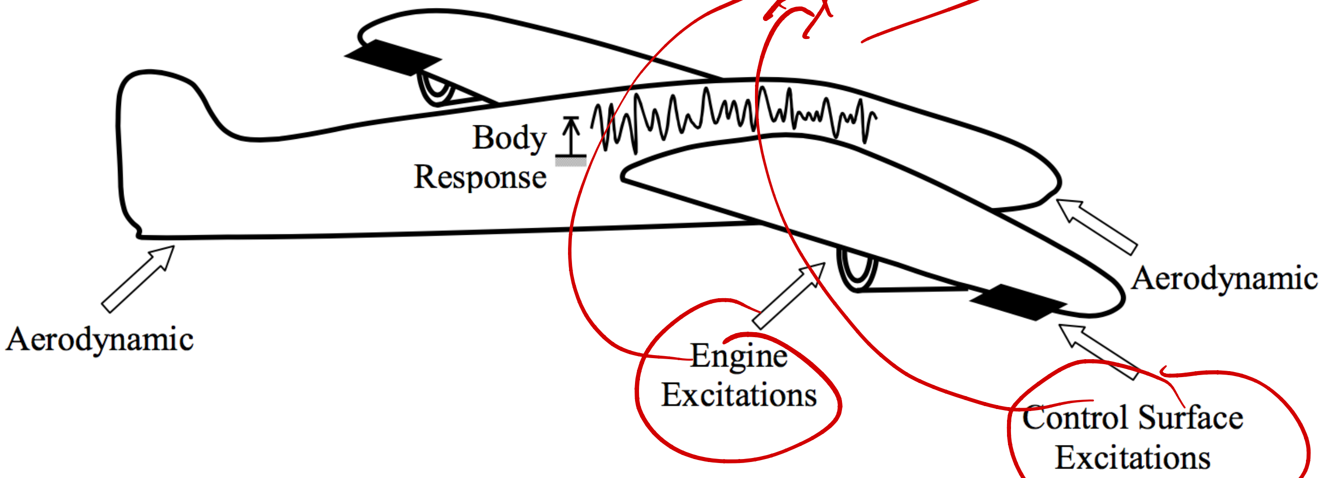
Foundations in Control Engineering

Make a **Dynamic System** behave in a desired manner, according to some **Performance Specifications**.

Modeling is an optimistic approach where we attempt to accurately represent the system or required system behavior

Note:

- Complex system
- Unknown excitations
- Unknown dynamics

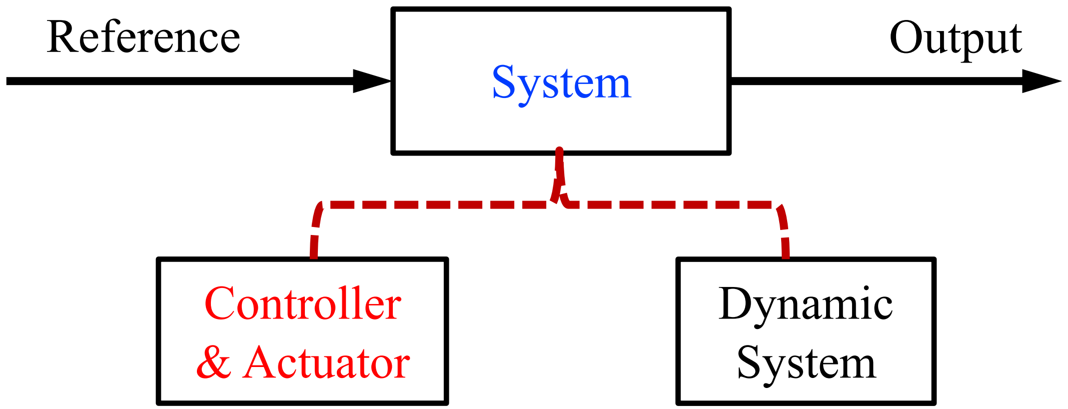


- Room temperature control
- Car/bicycle driving
- Balanced of bank account
- Laundry machine
- Airplane, rocket, satellite.

- DC motor

Use of Models

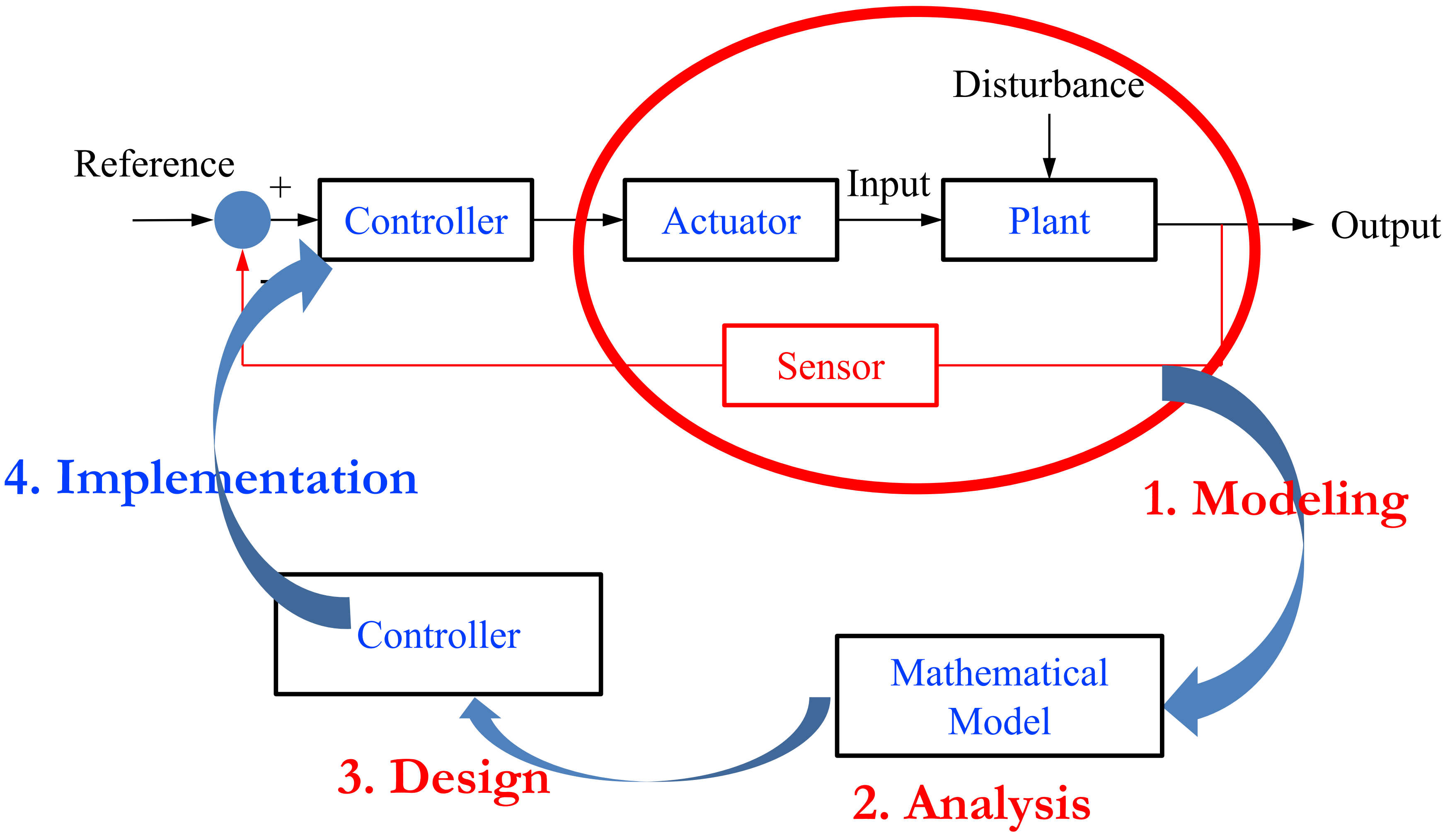
- **Initial Design**: The beginning of a design process (desired system does not exit)
- **Design Optimization**: design iteration, particularly prototyping can become very costly and time consuming
- **Monitoring and Fault Diagnosis**: identify faulty/degraded parts, provide suggestions
- **Design Evolution**: guide the design process by computer simulations



Engineering Examples:

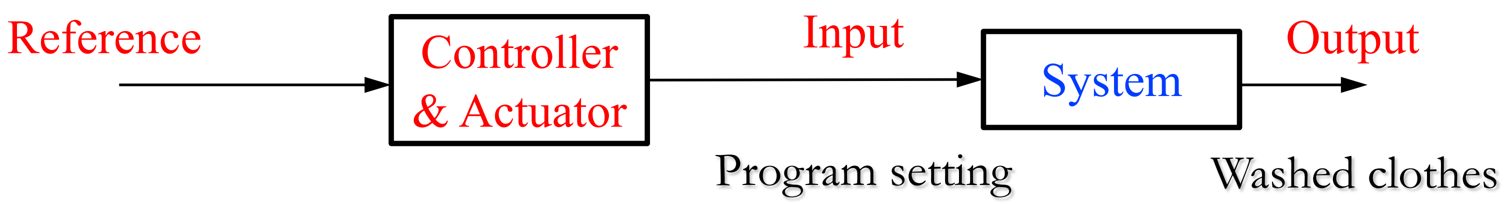
- ✓ Room temperature control
- ✓ Liquid-level control
- ✓ Steam pressure control
- ✓ Voice volume control

Systematic Controller Design Process



An Example: Laundry Machine

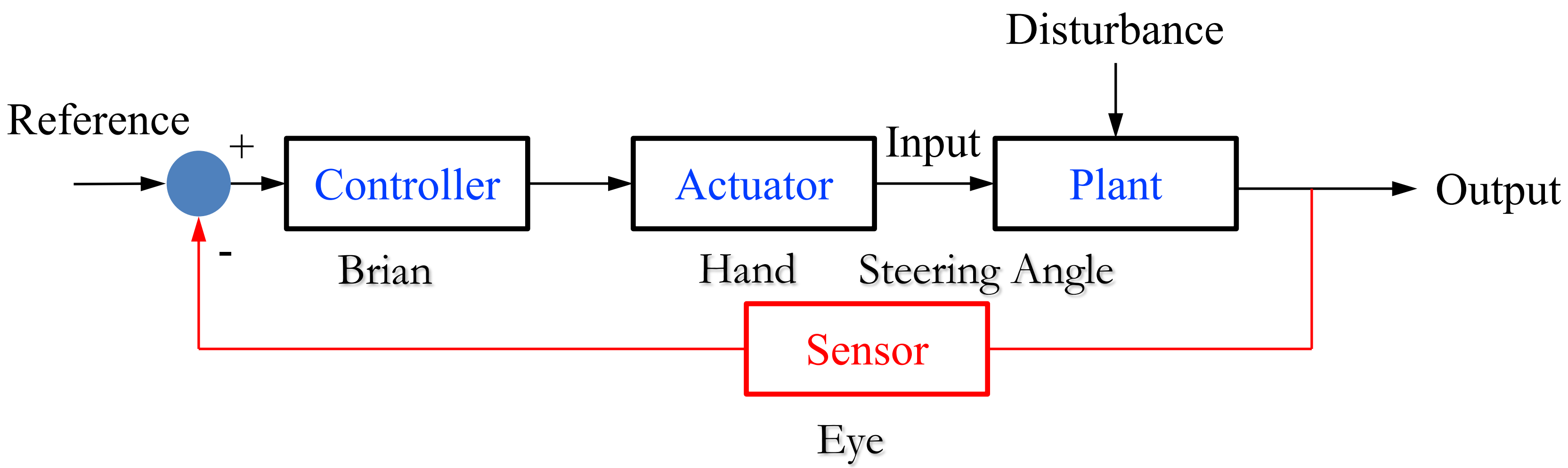
- A laundry machine washes clothes, by setting a program.



- A laundry machine does not measure how clean the clothes become.
- Control without measuring devices (sensors) are called ***open-loop control***.

An Example: Automobile

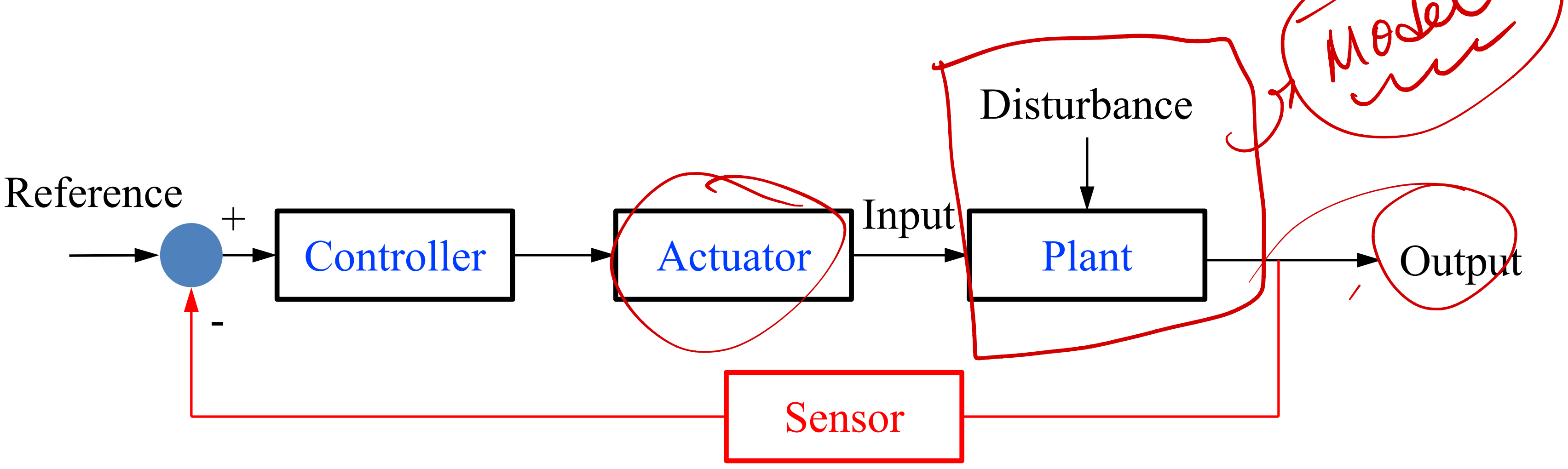
- Attempts to change the direction of the automobile.



- Manual closed-loop (**feedback**) control.
- Although the controlled system is "Automobile", the **input** and the **output** of the system can be different, depending on **control objectives**!

An Example: Automobile Cruise Control

- Attempts to maintain the speed of the automobile.



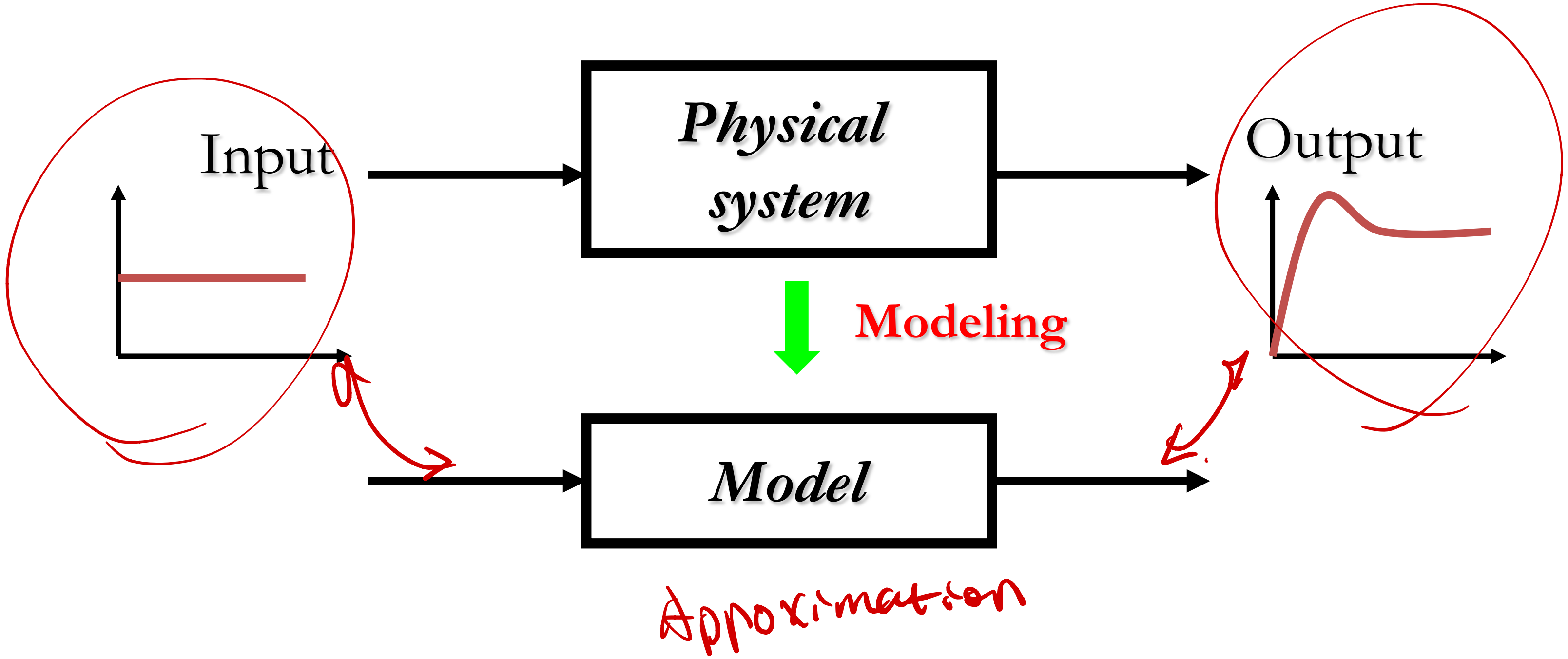
- Cruise control can be both manual and automatic.
- Note the similarity of the diagram above to the diagram in the previous slide!

Other Examples of Dynamic System

System	Typical Input	Typical Output
Human Body	Neuroelectric pulses	Muscle contraction, body movements
Company	Information	Decisions, finished products
Power plant	Fuel rate	Electric power, Pollution rate
Automobile	Steering wheel movement	Front wheel turn, direction of heading
Robot	Voltage to Joint	Joint motions, effector motion

Modeling (Analytical Model)

- Representation of the input-output relation of a physical system



- A model is used for the **analysis** and **design** of control systems.

Remarks on Models

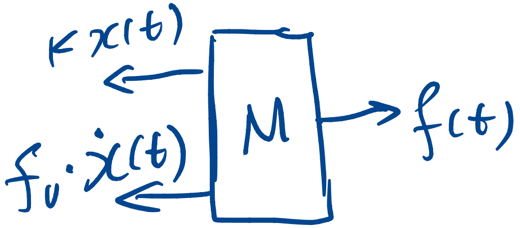
- Modeling is the most important and difficult task in control system design.
- No mathematical model exactly represents a physical system.

$$\text{Math model} \neq \text{Physical system}$$
$$\text{Math model} \approx \text{Physical system}$$

- Do not confuse models with physical systems!
- Selecting a model close enough to a physical system and yet simple enough to be studied analytically is the most important and difficult task.
- In this course, we may use the term "system" to mean a mathematical/analytical model.

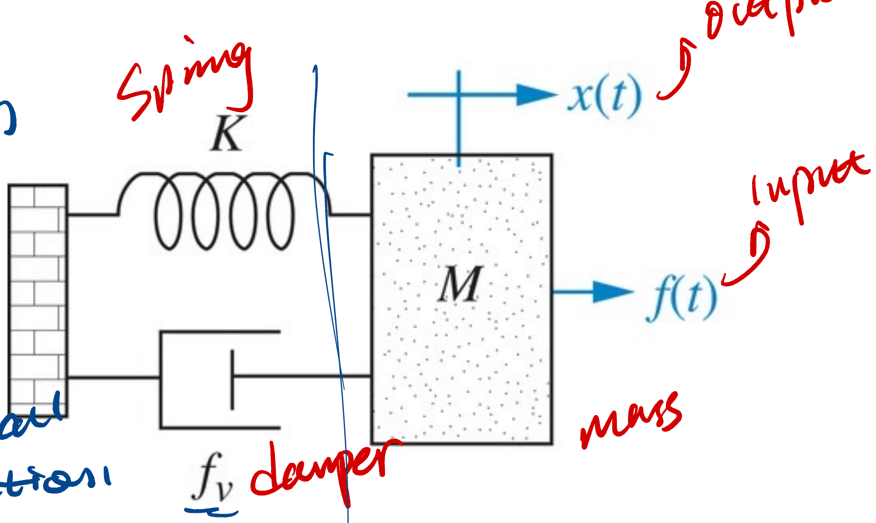
An Example

1. Draw FBD (Free Body Diagram)



Apply Newton's 2nd law of motion

$$\Sigma F = m\cdot a = M\ddot{x}(t)$$



$$f(t) - Kx(t) - f_v\cdot\dot{x}(t) = M\cdot\ddot{x}(t)$$

$$M\ddot{x}(t) + f_v\cdot\dot{x}(t) + Kx(t) = f(t)$$

differential equation

input

TF: input output

$$G(s) = \frac{\text{output}}{\text{input}}$$

Assuming zero initial conditions

$$M X(s)s^2 + f_v\cdot X(s)s + K\mathcal{L}(x) = F(s)$$

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}.$$

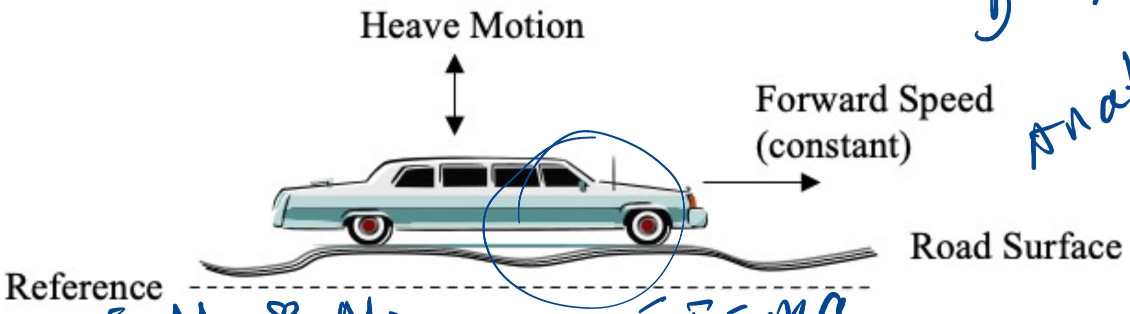
s domain. TF.

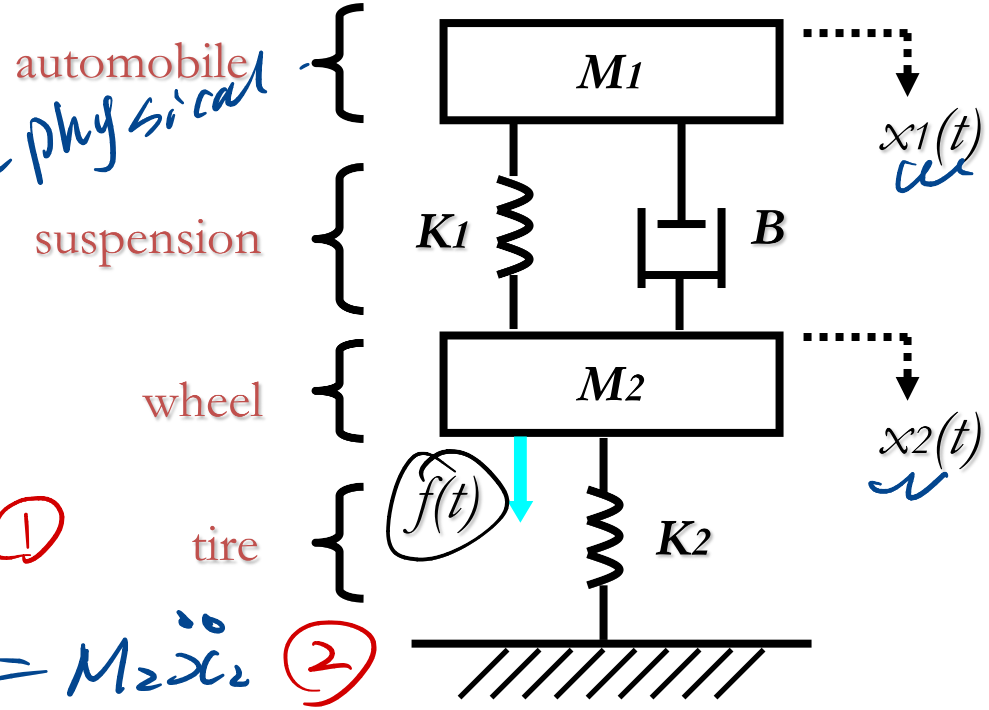
mass-spring-damper

Another Example

Modeling of automobile suspension system

accuracy.
simple.
analytical ≈ physical





FBD of M_1 & M_2 $\Sigma F = ma$

① $-K_1(x_1-x_2) - B(\dot{x}_1 - \dot{x}_2) = M_1\ddot{x}_1$

② $-K_2 x_2 + f(t) + K_1(x_1-x_2) + B(\dot{x}_1 - \dot{x}_2) = M_2\ddot{x}_2$

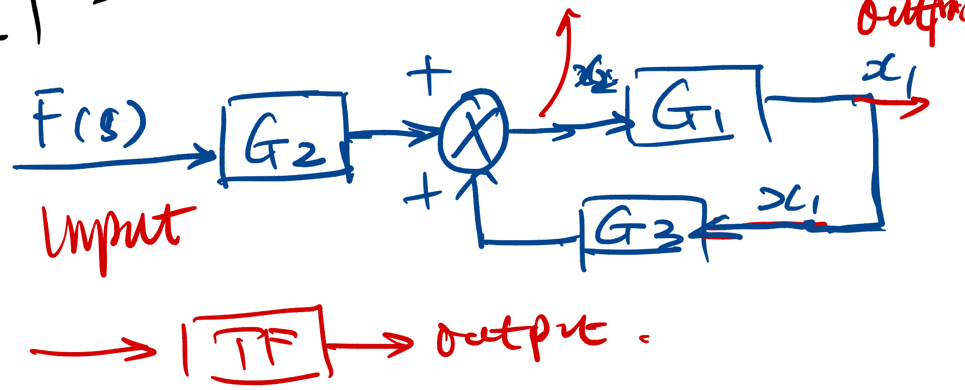
TF: $X_1 = \frac{Bs+K_1}{M_1 s^2 + Bs + K_1} \cdot X_2$ (G_1)

$X_1 = \frac{1}{M_2^2 s^2 + Bs + K_1 + K_2} F(s) + \frac{Bs + K_1}{M_2 s^2 + Bs + K_1 + K_2} X_2$ (G_2, G_3)

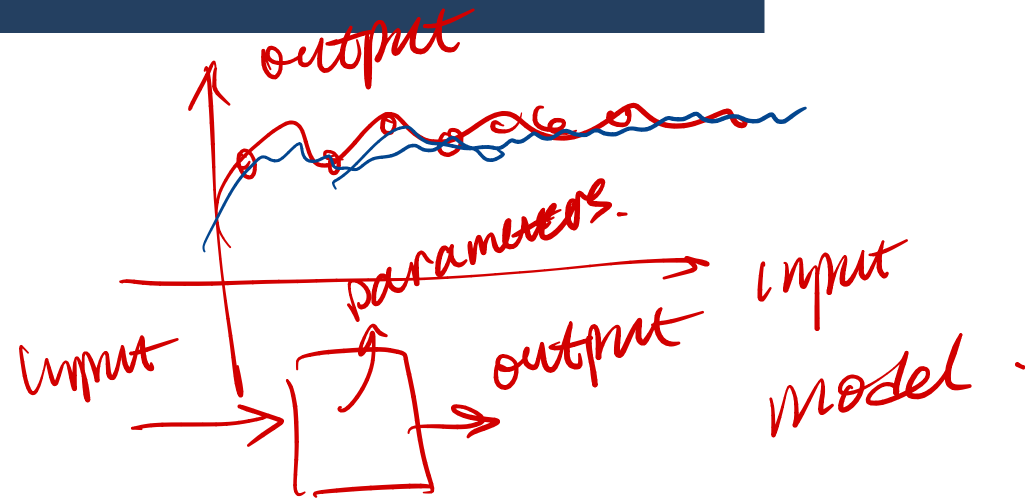
Block Diagram

Input → TF → output

$F(s)\cdot G_2 + G_3 x_1$



Model Types



Model: A "representation" of a system

Types of Models:

- Physical Models (Prototypes)
- Analytical Models
- Computer (Numerical) Models (Data Tables, Curves, Programs, Files, etc.)
- Experimental Models (use input/output experimental data for model "identification")

Dynamic System: Response variables are functions of time, with non-negligible rates of changes about an operating condition.

Note the implication of "approximation" in modeling

Model Complexity

Universal Model (which considers all aspects of the system) is unrealistic

E.g.: An automobile model that represents ride quality, energy consumption, traction characteristics, handling, structural strength, capacity, load characteristics, cost, safety, control, etc. can be complex and impractical

A model may address a few specific aspects of interest/application

Model should be as simple as possible (Approximate modeling, model reduction, etc. are applicable here)

Advantages of Analytical Models over Physical Models

- Modern, high-capacity, high-speed computers can accommodate complex analytical models
- Models can be modified quickly, and conveniently, at low cost
- High flexibility of making structural and parametric changes
- Naturally amenable to computer simulation
- Can be integrated with computer/numerical/experimental/ hardware models
- Can be done well before a prototype is built (and can be instrumental in deciding whether to prototype)

Note: Can't be easily understand and modified (an abstraction of the physical system)

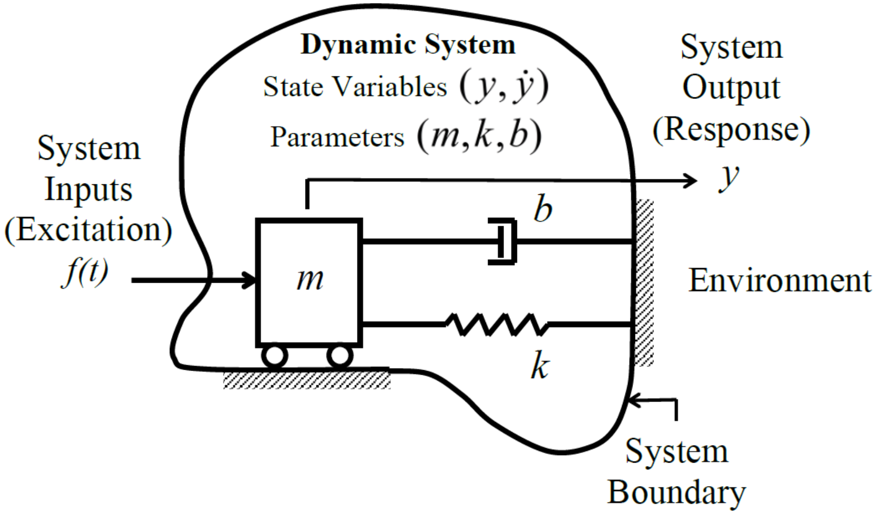
Analytical Model Types

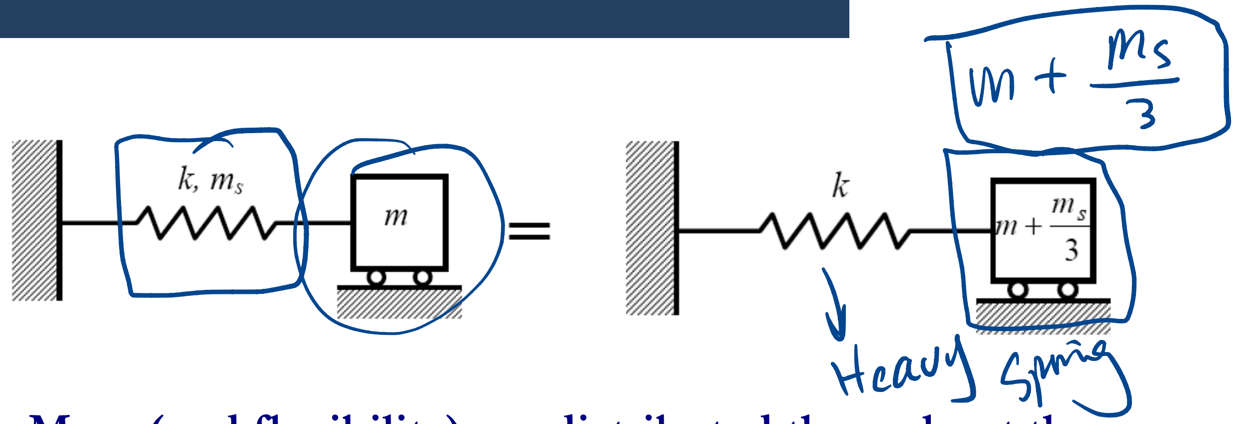
- **Nonlinear:** Nonlinear differential equations (**principle of superposition** does not hold)
- **Linear:** Linear differential equations (principle of superposition holds)
- **Distributed (Continuous)-parameter:** Partial differential equations (Dependent variables are functions of **time and space**)

- **Lumped-parameter:** Ordinary differential equations (Dependent variables are functions of time, not space)
- **Time-varying (Non-stationary, Non-autonomous):** Differential equations with time-varying coefficients (Model parameters vary with time)
- **Time-invariant (Stationary, Autonomous):** Differential equations with constant coefficients (Model parameters are constant)
- **Random (Stochastic):** Stochastic differential equations (Variables and/or parameters are governed by **probability distributions**)
- **Deterministic:** Non-stochastic differential equations (not governed by probabilities—repeat test under same conditions→ same results)
- **Continuous-time:** Differential equations (**Time** variable is **continuously** defined)
- **Discrete-time:** Difference equations (Time variable is defined as **discrete** values at a sequence of time points)

computer

An Example





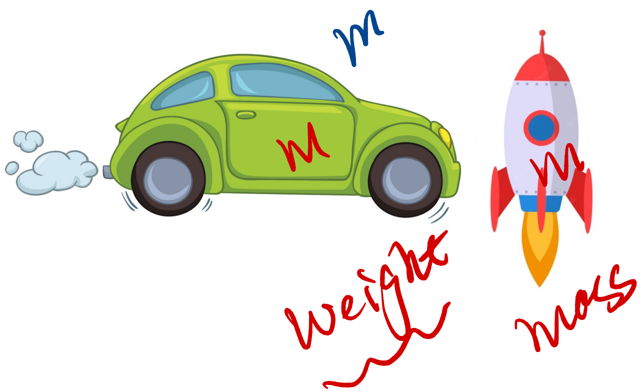
Mass (and flexibility) are distributed throughout the spring (not located at just a few discrete points)

$$m\ddot{y} + b\dot{y} + ky = f(t)$$

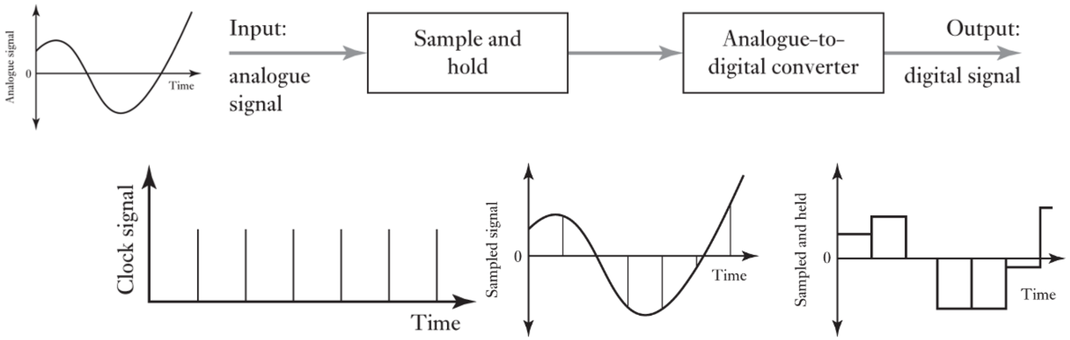
Nonlinear Damping

$$m\ddot{y} + c(\dot{y})\dot{y}^2 + ky = f(t)$$

Time varying



Analog to Digital Conversion



Differential Equations to Difference Equations

Summery

- Modeling: It is a representation of a dynamic system
- Useful in analysis, simulation, design, modification, control/operation, and evaluation/testing (e.g., qualification)
- An engineering (e.g., Mechatronic) physical system consists of a mixture of different types of components
- An engineering (e.g., Mechatronic) system is typically a multi-domain (or multi-physics or mixed) system

Next:

- Integrated: All domains are modeled together (concurrent)
- Unified: Use analogous procedures to model all components (in analytical modeling)
- Analogies exist in mechanical, electrical, fluid, and thermal systems

The End!!